

# Exploring Galaxy Clusters: Gravitational Lensing and (Dis)formal Non-Minimal Coupling

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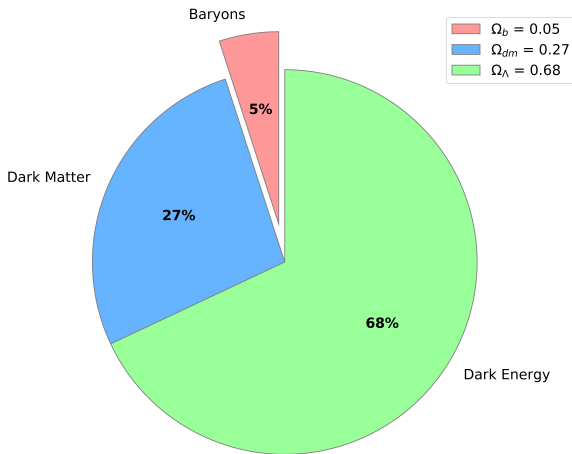
Institute of Physics, University of Szczecin, Poland  
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# DARK MATTER



95% of the Energy-Mass budget of the Universe:  
Dark matter + Dark Energy

# DARK MATTER

- Fritz Zwicky: 1930s - studied the Galaxy clusters  $\Rightarrow$  abnormal behaviour in speed of galaxies in the Coma cluster:
  - Galaxies were moving too fast based on just having visible matter
  - He suggested  $\Rightarrow$  “Dunkle Materie” or “dark matter”

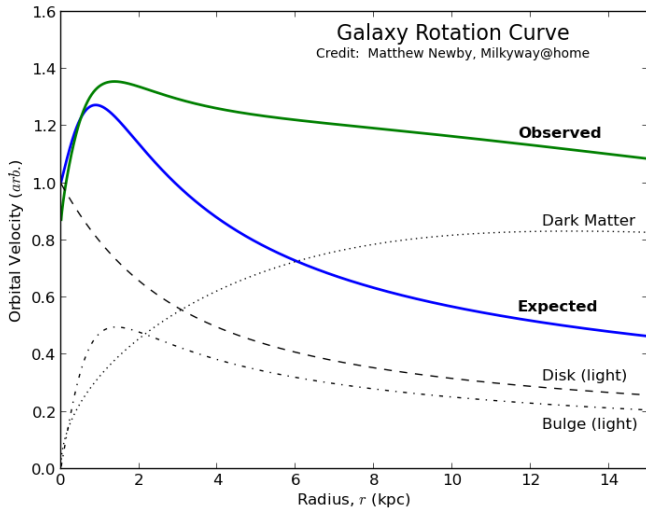


- Vera Rubin: 1970's - studied the speed of stars in galaxies:
  - Distant stars moving faster than we expected



*Vera Rubin ca 1992, photograph by Mark Godfrey, credit: Carnegie Institution, Department of Terrestrial Magnetism.*

# DARK MATTER



# DARK MATTER: OBSERVATIONAL EVIDENCE

- Rotational curve
- Mass tracers (X-rays, Sunyaev–Zeldovich, strong & weak lensing)
- Bullet Cluster
- Cosmic Microwave Background (CMB) anisotropies
- Large Scale Structure (LSS)



Credits: ESA

## $\Lambda$ CDM: $\Lambda$ -Cold Dark Matter

- **Abundance**

DM  $\Rightarrow$   $\sim 27\%$  of the universe's total mass-energy budget

- **Cold**

DM particles should be non-relativistic (cold)

- **Weakly Interacting**

DM is thought to interact via gravity and the weak nuclear force

- **Invisible**

It does not interact with light

- **Distribution**

A web-like structure throughout the universe

## NMC is not limited to specific theory

- Within GR, DM can be considered as: Bose-Einstein condensate (BEC)
- Modified Gravity  $\Rightarrow$  Born-Infeld theory  $\Rightarrow$  Same characteristics that NMC introduces (J. B. Jimenez, 2017, 1704.03351)



Einstein-Hilbert Action:

$$S = \int d^4x \sqrt{-g} R + S_{\text{fluid}} \quad (1)$$

Our NMC Action: (D. Bettoni, S. Liberati, 2015, 1502.06613)

$$S = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} [R + \alpha_{\text{d}} \rho_{\text{DM}} \mathbf{R}_{\mu\nu} \mathbf{u}^\mu \mathbf{u}^\nu] + S_{\text{fluid}} \quad (2)$$

Our DM variable couple to the contracted Ricci tensor with the fluid four-vector velocity.

Modified Poisson equation:

$$\nabla^2 \Phi = 4\pi G_N [\rho - \epsilon L^2 \nabla^2 \rho_{\text{DM}}] \quad (3)$$

The source of gravity is not only density but also on how the matter is distributed.

- Density  $\rho = \rho_{\text{DM}} + \rho_{\text{gas}}$
- Polarity  $\Rightarrow \epsilon = -1$
- Characteristic Length of NMC model  $\Rightarrow L \propto \alpha_d$

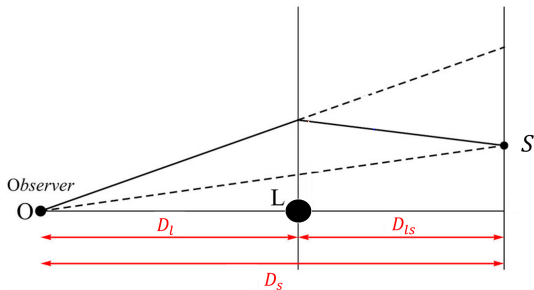
Modified Poisson equation:

$$\nabla^2 \Phi = 4\pi G_N [\rho - \epsilon L^2 \nabla^2 \rho_{\text{DM}}] \quad (4)$$

$$\nabla^2 \Psi_{ij} = 4\pi G \eta_{ij} [\rho - \epsilon L^2 \nabla^2 \rho_{\text{DM}}] \quad (5)$$

- $\Phi = \Psi \Rightarrow$  no anisotropic stress
- Density  $\rho = \rho_{\text{DM}} + \rho_{\text{gas}}$
- Polarity  $\Rightarrow \epsilon = -1$
- characteristic Length of NMC model  $\Rightarrow L$

# GRAVITATIONAL LENSING

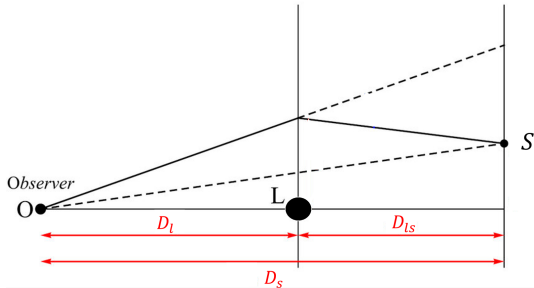


Effective lensing potential:

$$\Phi_{\text{lens}}(R) = \frac{2}{c^2} \frac{D_{ls}}{D_l D_s} \int_{-\infty}^{+\infty} \Phi(R, z) dz \quad (6)$$

$$\kappa(R) = \frac{1}{c^2} \frac{D_{ls} D_l}{D_s} \int_{-\infty}^{+\infty} \Delta_r \Phi(R, z) dz \quad (7)$$

# GRAVITATIONAL LENSING



$$\kappa(R) = \frac{4\pi G_N}{c^2} \frac{D_{ls}D_l}{D_s} \int_{-\infty}^{+\infty} [\rho(R, z) - \epsilon L^2 \Delta_r \rho_{DM}(R, z)] dz \quad (8)$$

$$\Delta_r = \frac{2}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} \text{ (Spherical coordinates)}$$

CLASH<sup>1</sup> survey programme: 19 clusters (Postman et al. 2011, 1106.3328)

- X-ray  $\Rightarrow$  hot gas
- strong and weak gravitational lensing  $\Rightarrow$  DM  $\Rightarrow$  NFW model

Hot gas = modified  $\beta$ -model (Donahue M. et al. 2015, 1405.7876)

$$\rho_{gas}(r) = \rho_{e,0} \left( \frac{r}{r_0} \right)^{-\alpha} \left[ 1 + \left( \frac{r}{r_{e,0}} \right)^2 \right]^{-3\beta_0/2} + \rho_{e,1} \left[ \left( \frac{r}{r_{e,1}} \right)^2 \right]^{-3\beta_1/2}$$

$\{\rho_{e,0}, r_{e,0}, r_{e,1}, r_0, \alpha, \beta_0, \beta_1\}$  fixed by preliminary fit of X-ray data

- Tension between X-ray and lensing data  $\Rightarrow$  Not including X-ray data directly in modeling our cluster

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<sup>1</sup>Cluster Lensing And Supernova survey with Hubble

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- strong and weak gravitational lensing  $\Rightarrow$  DM  $\Rightarrow$  NFW model

DM Model  $\Rightarrow$  Navarro-Frenk-White Profile (J. F. Navarro, C. S. Frenk, and S. D.

M. White, 1996)

$$\rho_{\text{NFW}}(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

$$\rho_s = \frac{\Delta}{3} \frac{c_\Delta^3}{\log(1 + c_\Delta) - \frac{c_\Delta}{1+c_\Delta}} \rho_c, \quad c_\Delta = \frac{r_\Delta}{r_s}, \quad \Delta = 200$$

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<sup>1</sup>Cluster Lensing And Supernova survey with Hubble

- $\chi^2$  function

$$\chi^2 = (\boldsymbol{\kappa}^{theo}(\boldsymbol{\theta}) - \boldsymbol{\kappa}^{obs}) \cdot \mathbf{C}^{-1} \cdot (\boldsymbol{\kappa}^{theo}(\boldsymbol{\theta}) - \boldsymbol{\kappa}^{obs}) \quad (9)$$

$\boldsymbol{\theta} = \{c_{200}, M_{200}, L\}$ : NMC model parameters

$\mathbf{C}$ : Covariance error matrix

- Bayes Theorem - Deriving the Posterior Distribution using MCMC

$$\mathcal{P}(\boldsymbol{\theta}, \mathcal{M}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta}, \mathcal{M})\pi(\boldsymbol{\theta}, \mathcal{M})}{\mathcal{E}(D|\mathcal{M})}$$

- Bayesian Evidence

$$\mathcal{E}(D|\mathcal{M}) = \int d\boldsymbol{\theta} \mathcal{L}(D|\boldsymbol{\theta}, \mathcal{M})\pi(\boldsymbol{\theta}, \mathcal{M})$$



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$$\mathcal{E}(D|\mathcal{M}) = \int d\boldsymbol{\theta} \mathcal{L}(D|\boldsymbol{\theta}, \mathcal{M}) \pi(\boldsymbol{\theta}, \mathcal{M})$$

- Bayes Factor

$$\mathcal{B}_j^i = \frac{\mathcal{E}(M_i)}{\mathcal{E}(M_j)}$$

- Jeffrey Scale  $\implies \mathcal{B}_j^i$  interpretation

$\log \mathcal{B}_j^i < 1$	Disfavoured
$1 \leq \log \mathcal{B}_j^i < 2.5$	Substantial
$2.5 \leq \log \mathcal{B}_j^i < 5$	Strong
$\log \mathcal{B}_j^i \geq 5$	Decisive

- Evidence: Highly prior dependent (Nisseris & Bellido 2013, 1210.7652)
- Kullback-Leibler divergence (KL) (Kullback & Leibler 1951)

$$\mathcal{D}_{KL,i} = \int d\theta \frac{\mathcal{L}(d|\theta, \mathcal{M}_i)}{\mathcal{E}(d|\mathcal{M}_i)} \log \frac{\mathcal{L}(d|\theta, \mathcal{M}_i)}{\mathcal{E}(d|\mathcal{M}_i)}$$

$\mathcal{D}_{KL,i}$ : prior-dependent as  $\mathcal{B}_{ij}$

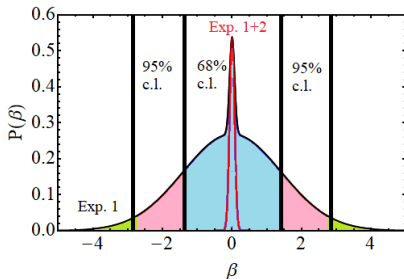
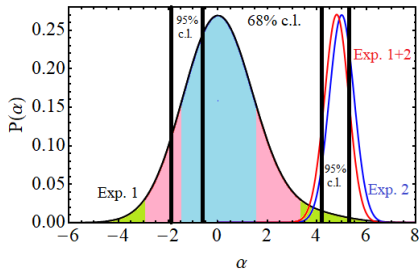
- Suspiciousness (Handley & Lemos 2019, 1903.06682, Handley & Lemos 2019, 1902.04029, Joackimi et al. 2021, 2102.09547)

$$\log \mathcal{S}_{ij} = \log \mathcal{B}_{ij} + \mathcal{D}_{KL,i} - \mathcal{D}_{KL,j}$$

$\log \mathcal{S}_{ij}$  prior-independent

$\log \mathcal{S}_{ij} < 0$	Tension
$\log \mathcal{S}_{ij} > 0$	Consistency

# WHEN MARGINALISATION IS NOT ENOUGH: AN EXAMPLE



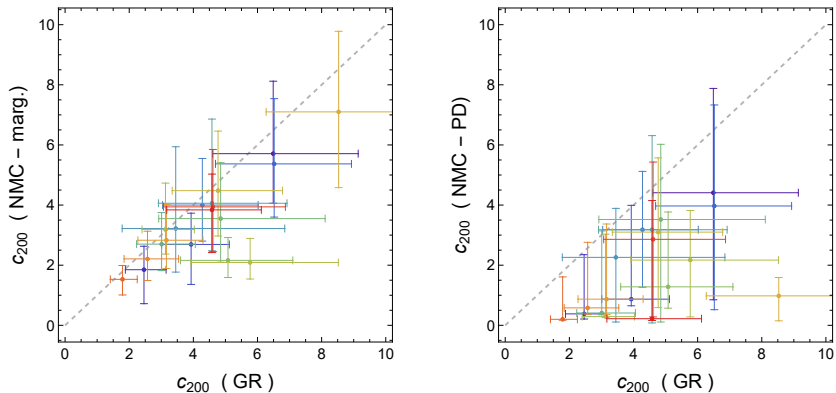
- **Marginalisation:**

- Simply “reading” the posteriors which are produced as output by the MCMCs.

- **Profile distribution:**

- An extension of the profile likelihood
- It highlights the behavior of the posterior around the maximum of the likelihood. (A. Gómez-Valent, 2022, 2203.16285)

# RESULTS: GR VS MODIFIED



**Figure 1:** Comparison between values of concentration parameter  $c_{200}$  in GR and NMC model considering Marg. and PD.

# RESULTS: GR VS MODIFIED

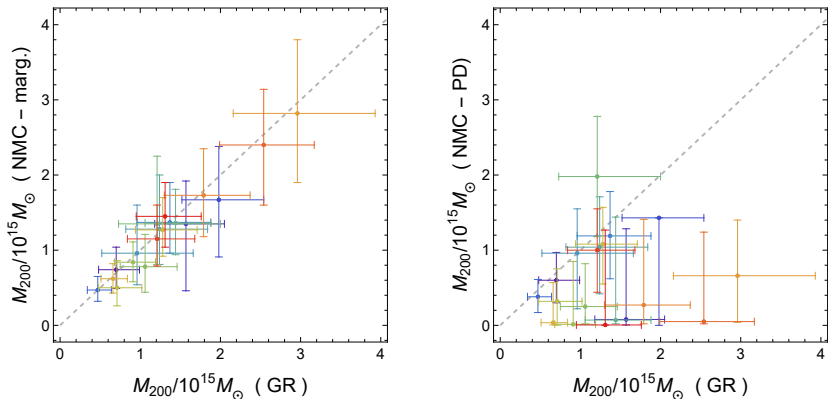


Figure 2: Comparison between values of mass  $M_{200}$  in GR and NMC model considering Marg. and PD.

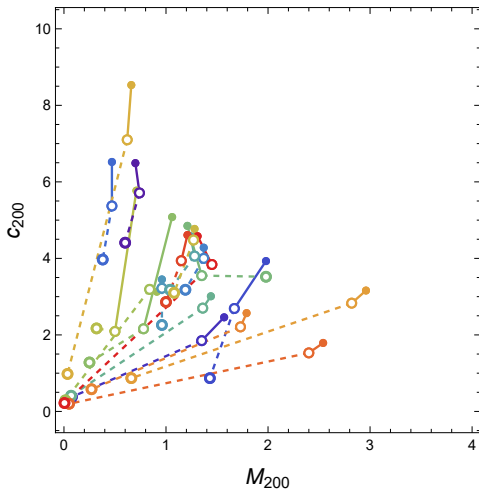
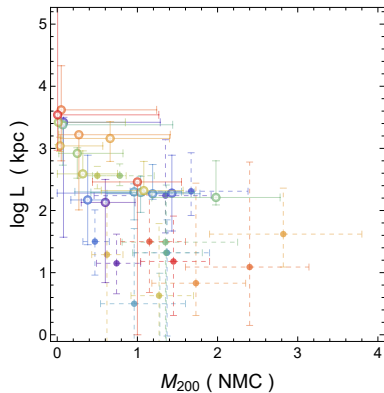
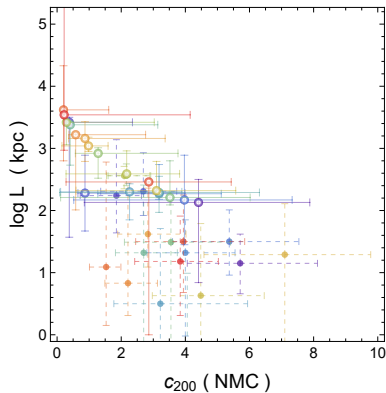


Figure 3: Comparison of dark matter parameters  $c_{200}$ ,  $M_{200}$ , obtained from GR and from the NMC model considered in this work.

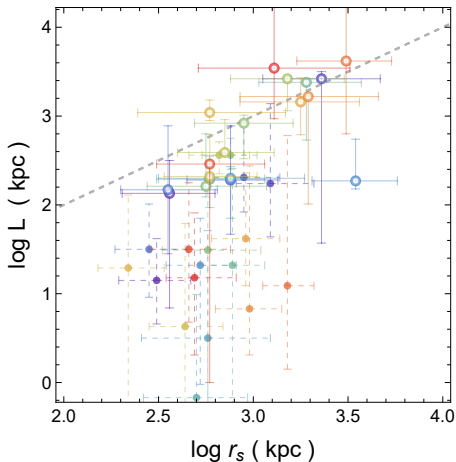
# RESULTS



$$\nabla^2 \Phi = 4\pi G_N [\rho - \epsilon L^2 \nabla^2 \rho_{\text{NFW}}]$$

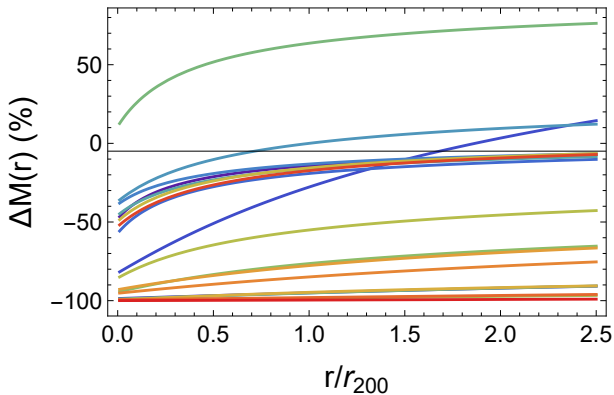
$$\rho_{\text{NFW}}(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}, \quad \rho_s = \frac{\Delta}{3} \frac{c_\Delta^3}{\log(1 + c_\Delta) - \frac{c_\Delta}{1+c_\Delta}} \rho_c$$

## RESULTS: CLOSER LOOK!



$$\rho_{\text{NFW}}(x) \propto \frac{1}{x(1+x)^2} + \frac{L^2}{r_s^2} \frac{6}{x(1+x)^4}, \quad (x = r/r_s)$$





$$\Delta M_{200} \equiv \frac{M_{200,\text{NMC}} - M_{200,\text{GR}}}{M_{200,\text{GR}}}$$

# Results

Cluster	GR		MOD (marg.)			MOD (PD)			$\log \mathcal{B}_j^i$	$\log \mathcal{S}_j^i$
	$c_{200}$	$M_{200}$ ( $10^{15} M_{\odot}$ )	$c_{200}$	$M_{200}$ ( $10^{15} M_{\odot}$ )	$\log L$ (kpc)	$c_{200}$	$M_{200}$ ( $10^{15} M_{\odot}$ )	$\log L$ (kpc)		
A383	$6.49^{+2.65}_{-1.89}$	$0.70^{+0.29}_{-0.22}$	$5.71^{+2.41}_{-1.65}$	$0.74^{+0.30}_{-0.25}$	$1.15^{+0.47}_{-0.49}$	$4.41^{+3.47}_{-3.56}$	$0.60^{+0.37}_{-0.30}$	(0.84, 2.13, 2.50)	$0.005^{+0.024}_{-0.023}$	$-0.010^{+0.035}_{-0.038}$
A209	$2.46^{+0.69}_{-0.58}$	$1.57^{+0.48}_{-0.39}$	$1.85^{+0.78}_{-1.13}$	$1.35^{+0.57}_{-0.89}$	$2.24^{+0.90}_{-0.60}$	(0.21, 0.38, 2.35)	(0.005, 0.078, 1.285)	(1.57, 3.42, 3.50)	$0.006^{+0.022}_{-0.022}$	$-0.025^{+0.038}_{-0.052}$
A2261	$3.93^{+1.19}_{-0.92}$	$1.98^{+0.56}_{-0.46}$	$2.69^{+1.04}_{-1.33}$	$1.67^{+0.71}_{-0.76}$	$2.31^{+0.62}_{-0.39}$	(0.65, 0.87, 3.99)	< 1.43	$2.28^{+0.61}_{-0.61}$	$-0.0005^{+0.0190}_{-0.0202}$	$0.011^{+0.034}_{-0.037}$
RXJ2129	$6.52^{+2.41}_{-1.83}$	$0.47^{+0.17}_{-0.13}$	$5.37^{+2.17}_{-1.77}$	$0.47^{+0.18}_{-0.15}$	$1.50^{+0.51}_{-0.54}$	$3.97^{+3.36}_{-3.45}$	$0.38^{+0.23}_{-0.21}$	$2.17^{+0.72}_{-0.72}$	$0.014^{+0.022}_{-0.023}$	$0.005^{+0.040}_{-0.037}$
A611	$4.28^{+1.74}_{-1.24}$	$1.37^{+0.51}_{-0.41}$	$4.00^{+1.55}_{-1.20}$	$1.37^{+0.53}_{-0.41}$	$1.32^{+0.53}_{-1.34}$	$3.18^{+1.94}_{-1.91}$	$1.19^{+0.59}_{-0.57}$	(2.18, 2.27, 2.74)	$-0.013^{+0.021}_{-0.022}$	$-0.021^{+0.046}_{-0.041}$
MS2137	$3.45^{+3.40}_{-1.67}$	$0.96^{+0.70}_{-0.44}$	$3.22^{+2.72}_{-1.45}$	$0.96^{+0.64}_{-0.42}$	$0.50^{+1.21}_{-2.49}$	(0.11, 2.26, 3.89)	$0.96^{+0.59}_{-0.74}$	(1.85, 2.30, 2.43)	$0.057^{+0.021}_{-0.025}$	$0.084^{+0.031}_{-0.043}$
RXJ2248	$4.58^{+2.34}_{-1.67}$	$1.24^{+0.60}_{-0.42}$	$4.06^{+2.80}_{-1.58}$	$1.28^{+0.72}_{-0.47}$	$-0.17^{+1.16}_{-1.73}$	$3.18^{+3.13}_{-3.10}$	$1.04^{+0.67}_{-0.62}$	(1.97, 2.29, 2.55)	$-0.049^{+0.017}_{-0.025}$	$-0.077^{+0.035}_{-0.044}$
MACSJ1115	$3.01^{+1.05}_{-0.78}$	$1.44^{+0.44}_{-0.38}$	$2.70^{+1.05}_{-0.88}$	$1.36^{+0.45}_{-0.42}$	$1.32^{+1.09}_{-2.44}$	(0.35, 0.41, 3.14)	(0.02, 0.07, 1.44)	(2.73, 3.38, 3.42)	$0.014^{+0.023}_{-0.022}$	$0.008^{+0.032}_{-0.037}$
MACSJ1931	$4.85^{+3.26}_{-1.93}$	$1.21^{+0.79}_{-0.48}$	$3.55^{+1.86}_{-1.45}$	$1.35^{+0.90}_{-0.54}$	$1.49^{+0.81}_{-2.44}$	(0.11, 3.52, 6.02)	$1.98^{+0.80}_{-0.80}$	(2.09, 2.21, 2.80)	$-0.025^{+0.027}_{-0.020}$	$-0.056^{+0.038}_{-0.037}$
MACSJ1720	$5.08^{+2.02}_{-1.48}$	$1.06^{+0.40}_{-0.31}$	$2.16^{+0.76}_{-0.57}$	$0.78^{+0.43}_{-0.34}$	$2.56^{+0.19}_{-0.16}$	(0.57, 1.28, 3.77)	(0.02, 0.25, 0.82)	(2.52, 2.92, 3.01)	$0.009^{+0.023}_{-0.022}$	$-0.016^{+0.046}_{-0.032}$
MACSJ0416	$3.13^{+0.90}_{-0.73}$	$0.91^{+0.28}_{-0.23}$	$3.19^{+1.54}_{-0.82}$	$0.84^{+0.27}_{-0.26}$	$-1.89^{+1.47}_{-3.43}$	(0.26, 0.29, 3.03)	(0.010, 0.014, 0.850)	(3.06, 3.42, 3.44)	$-0.073^{+0.026}_{-0.022}$	$-0.143^{+0.046}_{-0.037}$
MACSJ0429	$5.77^{+2.75}_{-1.85}$	$0.71^{+0.31}_{-0.23}$	$2.09^{+0.80}_{-0.55}$	$0.50^{+0.36}_{-0.24}$	$2.56^{+0.15}_{-0.21}$	(0.28, 2.17, 3.82)	(0.002, 0.319, 0.752)	(2.58, 2.59, 2.96)	$-0.083^{+0.020}_{-0.025}$	$-0.154^{+0.038}_{-0.046}$
MACSJ1206	$4.77^{+2.01}_{-1.43}$	$1.28^{+0.43}_{-0.34}$	$4.48^{+1.98}_{-1.51}$	$1.27^{+0.43}_{-0.35}$	$0.63^{+1.16}_{-1.29}$	$3.10^{+2.47}_{-2.51}$	$1.08^{+0.49}_{-0.53}$	(2.26, 2.32, 2.79)	$0.004^{+0.020}_{-0.022}$	$-0.0004^{+0.0355}_{-0.0402}$
MACSJ0329	$8.53^{+2.71}_{-2.26}$	$0.66^{+0.18}_{-0.15}$	$7.10^{+2.68}_{-2.52}$	$0.62^{+0.20}_{-0.19}$	$1.29^{+0.82}_{-2.30}$	$0.98^{+0.61}_{-0.83}$	(0.006, 0.035, 0.568)	(2.95, 3.04, 3.18)	$0.009^{+0.020}_{-0.024}$	$0.0004^{+0.0374}_{-0.0455}$
RXJ1347	$3.16^{+1.14}_{-0.89}$	$2.96^{+0.97}_{-0.80}$	$2.83^{+1.19}_{-0.94}$	$2.82^{+0.98}_{-0.92}$	$1.62^{+0.74}_{-0.53}$	(0.36, 0.87, 3.37)	(0.04, 0.66, 1.40)	(2.79, 3.16, 3.43)	$0.008^{+0.029}_{-0.026}$	$0.005^{+0.053}_{-0.045}$
MACSJ1149	$2.57^{+0.97}_{-0.73}$	$1.79^{+0.58}_{-0.49}$	$2.21^{+0.92}_{-0.72}$	$1.73^{+0.62}_{-0.55}$	$0.83^{+1.61}_{-0.52}$	(0.37, 0.58, 2.76)	(0.02, 0.27, 1.41)	(2.01, 3.22, 3.23)	$0.033^{+0.023}_{-0.021}$	$0.042^{+0.048}_{-0.029}$
MACSJ0717	$1.79^{+0.46}_{-0.38}$	$2.54^{+0.63}_{-0.55}$	$1.53^{+0.46}_{-0.42}$	$2.40^{+0.74}_{-0.80}$	$1.09^{+1.69}_{-0.94}$	(0.17, 0.20, 1.61)	(0.02, 0.05, 1.24)	$3.62^{+0.71}_{-0.82}$	$0.008^{+0.022}_{-0.017}$	$0.002^{+0.037}_{-0.030}$
MACSJ0647	$4.61^{+2.26}_{-1.54}$	$1.21^{+0.47}_{-0.37}$	$3.94^{+1.91}_{-1.49}$	$1.15^{+0.45}_{-0.36}$	$1.50^{+0.75}_{-0.82}$	$2.86^{+2.57}_{-2.58}$	$1.00^{+0.55}_{-0.56}$	> 2.46	$0.041^{+0.023}_{-0.020}$	$0.046^{+0.038}_{-0.034}$
MACSJ0744	$4.58^{+2.09}_{-1.41}$	$1.31^{+0.45}_{-0.36}$	$3.84^{+1.19}_{-1.12}$	$1.45^{+0.45}_{-0.41}$	$1.18^{+0.73}_{-0.87}$	(0.16, 0.22, 4.15)	(0.002, 0.005, 1.267)	(2.97, 3.54, 3.62)	$-0.002^{+0.021}_{-0.023}$	$-0.003^{+0.032}_{-0.048}$

# Results

Cluster	GR		MOD (marg.)			MOD (PD)			$\log \mathcal{B}_i^j$	$\log \mathcal{S}_i^j$
	$c_{200}$	$M_{200}$ ( $10^{15} M_{\odot}$ )	$c_{200}$	$M_{200}$ ( $10^{15} M_{\odot}$ )	$\log L$ ( $kpc$ )	$c_{200}$	$M_{200}$ ( $10^{15} M_{\odot}$ )	$\log L$ ( $kpc$ )		
A383	$6.49^{+2.65}_{-1.89}$	$0.70^{+0.29}_{-0.22}$	$5.71^{+2.41}_{-1.65}$	$0.74^{+0.30}_{-0.25}$	<b><math>1.15^{+0.47}_{-0.49}</math></b>	$4.41^{+3.47}_{-3.56}$	$0.60^{+0.37}_{-0.30}$	(0.84, 2.13, 2.50)	$0.005^{+0.024}_{-0.023}$	$-0.010^{+0.035}_{-0.038}$
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MS2137	$3.45^{+3.40}_{-1.67}$	$0.96^{+0.70}_{-0.44}$	$3.22^{+2.72}_{-1.45}$	$0.96^{+0.64}_{-0.42}$	$0.50^{+1.21}_{-0.15}$	(0.11, 2.26, 3.89)	$0.96^{+0.59}_{-0.74}$	(1.85, 2.30, 2.43)	$0.057^{+0.021}_{-0.025}$	$0.084^{+0.031}_{-0.043}$
RXJ2248	$4.58^{+2.34}_{-1.67}$	$1.24^{+0.60}_{-0.42}$	$4.06^{+2.80}_{-1.58}$	$1.28^{+0.72}_{-0.47}$	$-0.17^{+1.16}_{-1.73}$	$3.18^{+3.13}_{-3.10}$	$1.04^{+0.67}_{-0.62}$	(1.97, 2.29, 2.55)	$-0.049^{+0.017}_{-0.025}$	$-0.077^{+0.035}_{-0.044}$
MACSJ1115	$3.01^{+1.05}_{-0.78}$	$1.44^{+0.44}_{-0.38}$	$2.70^{+1.05}_{-0.88}$	$1.36^{+0.45}_{-0.42}$	$1.32^{+1.09}_{-2.44}$	(0.35, 0.41, 3.14)	(0.02, 0.07, 1.44)	(2.73, 3.38, 3.42)	$0.014^{+0.023}_{-0.022}$	$0.008^{+0.032}_{-0.037}$
MACSJ1931	$4.85^{+3.26}_{-1.93}$	$1.21^{+0.79}_{-0.48}$	$3.55^{+1.86}_{-1.45}$	$1.35^{+0.90}_{-0.54}$	$1.49^{+0.81}_{-0.54}$	(0.11, 3.52, 6.02)	$1.98^{+0.80}_{-0.80}$	(2.09, 2.21, 2.80)	$-0.025^{+0.027}_{-0.020}$	$-0.056^{+0.038}_{-0.037}$
MACSJ1720	$5.08^{+2.02}_{-1.48}$	$1.06^{+0.40}_{-0.31}$	$2.16^{+0.76}_{-0.57}$	$0.78^{+0.43}_{-0.34}$	$2.56^{+0.19}_{-0.16}$	(0.57, 1.28, 3.77)	(0.02, 0.25, 0.82)	(2.52, 2.92, 3.01)	$0.009^{+0.023}_{-0.022}$	$-0.016^{+0.046}_{-0.032}$
MACSJ0416	$3.13^{+0.90}_{-0.73}$	$0.91^{+0.28}_{-0.23}$	$3.19^{+1.54}_{-0.82}$	$0.84^{+0.27}_{-0.26}$	$-1.89^{+1.47}_{-3.43}$	(0.26, 0.29, 3.03)	(0.010, 0.014, 0.850)	(3.06, 3.42, 3.44)	$-0.073^{+0.026}_{-0.022}$	$-0.143^{+0.046}_{-0.037}$
MACSJ0429	$5.77^{+2.75}_{-1.85}$	$0.71^{+0.31}_{-0.23}$	$2.09^{+0.80}_{-0.50}$	$0.50^{+0.36}_{-0.24}$	$2.56^{+0.15}_{-0.21}$	(0.28, 2.17, 3.82)	(0.002, 0.319, 0.752)	(2.58, 2.59, 2.96)	$-0.083^{+0.020}_{-0.025}$	$-0.154^{+0.038}_{-0.046}$
MACSJ1206	$4.77^{+2.01}_{-1.43}$	$1.28^{+0.43}_{-0.34}$	$4.48^{+1.98}_{-1.51}$	$1.27^{+0.43}_{-0.35}$	$0.63^{+1.16}_{-1.23}$	$3.10^{+2.47}_{-2.51}$	$1.08^{+0.49}_{-0.53}$	(2.26, 2.32, 2.79)	$0.004^{+0.020}_{-0.022}$	$-0.0004^{+0.0355}_{-0.0402}$
MACSJ0329	$8.53^{+2.71}_{-2.26}$	$0.66^{+0.18}_{-0.15}$	$7.10^{+2.68}_{-2.52}$	$0.62^{+0.20}_{-0.19}$	$1.29^{+0.82}_{-2.30}$	$0.98^{+0.61}_{-0.83}$	(0.006, 0.035, 0.568)	(2.95, 3.04, 3.18)	$0.009^{+0.020}_{-0.024}$	$0.0004^{+0.0374}_{-0.0455}$
RXJ1347	$3.16^{+1.14}_{-0.89}$	$2.96^{+0.97}_{-0.80}$	$2.83^{+1.19}_{-0.94}$	$2.82^{+0.98}_{-0.92}$	$1.62^{+0.74}_{-0.53}$	(0.36, 0.87, 3.37)	(0.04, 0.66, 1.40)	(2.79, 3.16, 3.43)	$0.008^{+0.029}_{-0.026}$	$0.005^{+0.053}_{-0.045}$
MACSJ1149	$2.57^{+0.97}_{-0.73}$	$1.79^{+0.58}_{-0.49}$	$2.21^{+0.92}_{-0.72}$	$1.73^{+0.62}_{-0.55}$	$0.83^{+1.61}_{-0.52}$	(0.37, 0.58, 2.76)	(0.02, 0.27, 1.41)	(2.01, 3.22, 3.23)	$0.033^{+0.023}_{-0.021}$	$0.042^{+0.048}_{-0.029}$
MACSJ0717	$1.79^{+0.46}_{-0.38}$	$2.54^{+0.63}_{-0.55}$	$1.53^{+0.46}_{-0.52}$	$2.40^{+0.74}_{-0.80}$	$1.09^{+1.69}_{-0.94}$	(0.17, 0.20, 1.61)	(0.02, 0.05, 1.24)	$3.62^{+0.71}_{-0.82}$	$0.008^{+0.022}_{-0.017}$	$0.002^{+0.037}_{-0.030}$
MACSJ0647	$4.61^{+2.26}_{-1.54}$	$1.21^{+0.47}_{-0.37}$	$3.94^{+1.91}_{-1.49}$	$1.15^{+0.45}_{-0.36}$	$1.50^{+0.75}_{-0.82}$	$2.86^{+2.57}_{-2.58}$	$1.00^{+0.55}_{-0.56}$	> 2.46	$0.041^{+0.023}_{-0.020}$	$0.046^{+0.038}_{-0.034}$
MACSJ0744	$4.58^{+2.09}_{-1.41}$	$1.31^{+0.45}_{-0.36}$	$3.84^{+1.19}_{-1.42}$	$1.45^{+0.45}_{-0.41}$	$1.18^{+0.73}_{-0.87}$	(0.16, 0.22, 4.15)	(0.002, 0.005, 1.267)	(2.97, 3.54, 3.62)	$-0.002^{+0.021}_{-0.023}$	$-0.003^{+0.032}_{-0.048}$

## SUMMARY & CONCLUSION

- Disformally NMC Dark matter
- Modified Poisson equation  $\Rightarrow$  depend to gradient of the density
- CLASH  $\Rightarrow$  NFW + Modified  $\beta$ -model
- Volume effects  $\Rightarrow$  Narrow minimum  $\Rightarrow$  Marg. + PD

- Marginal analysis results  $\Rightarrow$  more close to GR
- $L \Rightarrow 0.1 - 10^2$  kpc  $\Rightarrow$  average value  $\sim 10$  kpc

- PD:  $c_{200}, M_{200} < \text{GR}$
- Marginalised to PD  $\Rightarrow L \ll r_s$  to  $L \sim r_s$



THANK YOU!

